Minimal models 14 - signs (checked)

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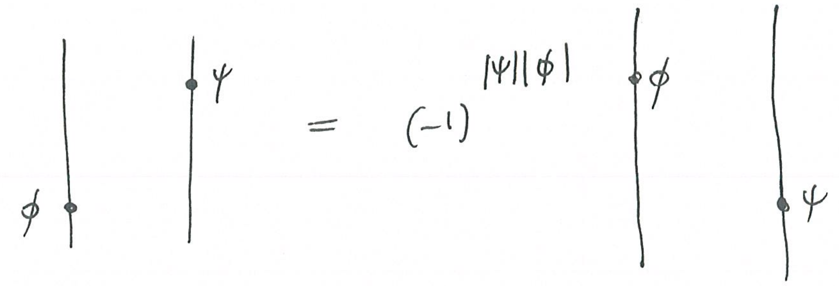
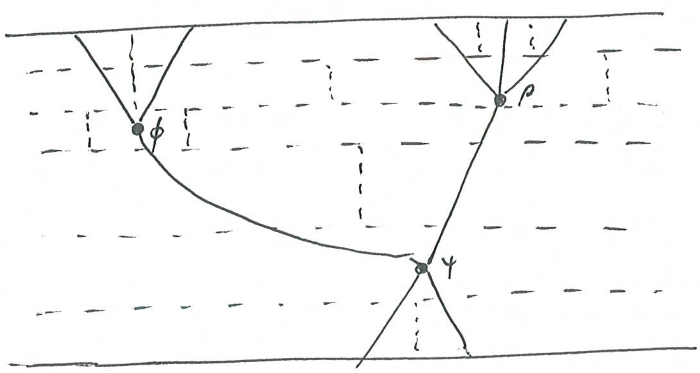
Here we clarify some issues with signs, trees and operators assigned to trees. Let k be a commutative ring, $\mathcal{V}$ the symmetric monoidal category of $\mathbb{Z}_2$-graded k-modules. We have the standard diagrammatic calculus for the category where morphisms are degree zero maps (this is what we mean by $\mathcal{V}$), but when we start drawing diagrams with nonzero degree maps at vertices, we have to account for

$$\phi \otimes \psi := (\phi \otimes 1)(1 \otimes \psi) = (-1)^{|\psi||\phi|}(1 \otimes \psi)(\phi \otimes 1)$$



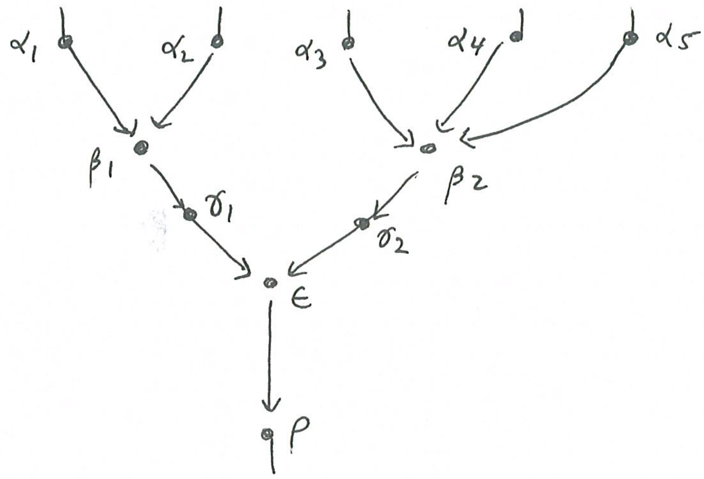
(1-1)

Here to match trees in A_∞-theory we read diagrams from top to bottom and left to right. In particular this identity means we never allow vertices on the same height coordinate. So, consider all progressive planar diagrams, generic in that they have no two vertices at the same height, where we allow vertices to have labels which are nonzero degree. Then in the usual way we may define the valuation of such a diagram



$$\psi \circ (\phi \otimes 1) \circ (1 \otimes \rho). \tag{1.2}$$

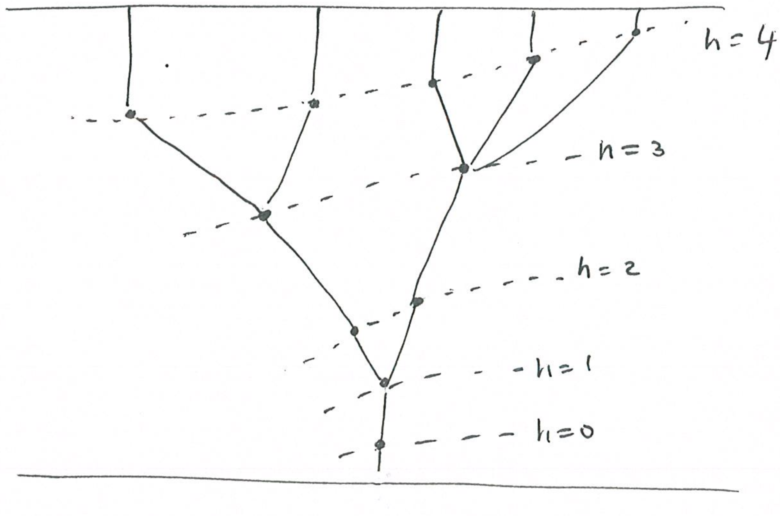
Now suppose given an abstract (i.e. not embedded) tree T which is planar (a property of orientation at vertices) and oriented, with labels on all its vertices which are homogeneous maps of $\mathbb{Z}_2$-graded k-modules. For example



(2.1)

Here the leaves and root are resp. morphisms out of (into) a fixed $\mathbb{Z}_2$-graded module V. We may assign this abstract tree an embedding into a strip $[a,b] \times \mathbb{R}$ as in (1.2) in various ways, which yield the same linear map as the valuation - <u>up to a sign</u>. This sign will be fixed once we assign a <u>height</u> to each vertex of the tree.

The height $h(v) \in \mathbb{R}$ of a vertex v is the <u>length of the unique path to the root, with ties broken by orientation</u> (i.e. reading left to right in any compatible planar presentation (2.1)). So (2.1) becomes



(2.2)

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(3)

Hence the operator associated to (2.1) is

$$\rho \circ \epsilon \circ (\gamma_1 \otimes \gamma_2) \circ (\beta_1 \otimes \beta_2) \circ (\alpha_1 \otimes \alpha_2 \otimes \alpha_3 \otimes \alpha_4 \otimes \alpha_5). \tag{3.1}$$