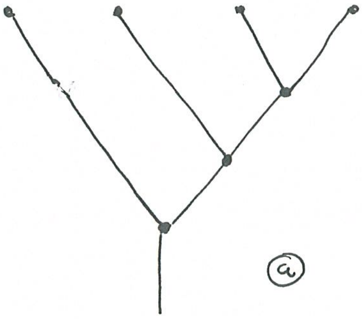
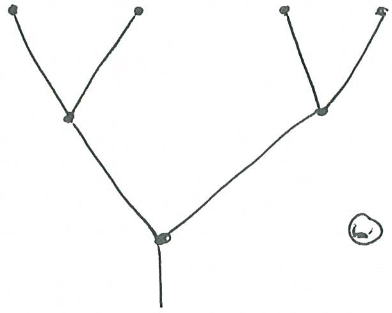
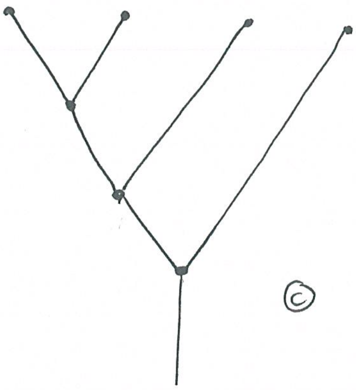
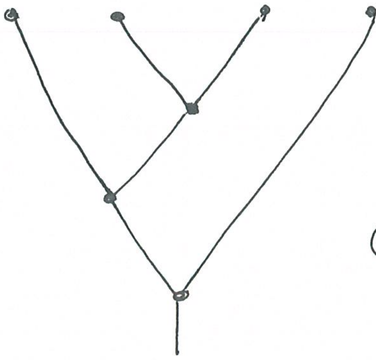
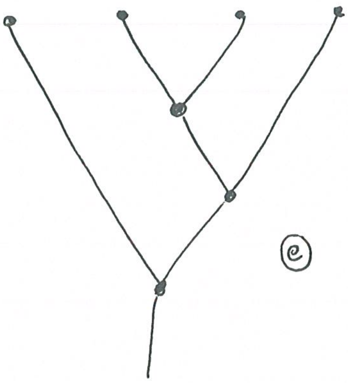
Minimal models for MFs VI (checked)

We continue the calculation of the minimal model for $W = y^d - x^d$ begun on p. 5 of ainfmf5. First though we describe b_4 in general from p. 3 of ainfmf2,

$b_4 =$  (a) $+$  (b) (1.1)

$+$  (c) $+$  (d)

$+$  (e)

Back in the case of $W = y^d - x^d$ $d > 2$ we compute b_4 on the $\mathbb{Z}_2$-graded vector space $\mathcal{A} = \Lambda(k\psi_1^* \oplus k\psi_2^*)$ of (6.3), from using (7.1) there (ainfmf6) (ainfmf5)

$$\delta_\infty = \delta - x^{d-2}[\psi_1, -]\theta_1\delta + y^{d-2}[\psi_2, -]\theta_2\delta \qquad (2.1)$$
$$+ x^{d-2}y^{d-2}[\psi_1,-][\psi_2,-]\theta_1\theta_2\delta$$

and (8.1) there, where $p = |\omega|$, $b = |f|$

$$H_\infty(\omega \otimes f\Psi) = \frac{1}{p+b}\left\{\partial_x(f)\theta_1 + \partial_y(f)\theta_2\right\}(\omega\otimes\Psi) \qquad (2.2)$$
$$- \frac{(d-1)}{(p+b)(p+b+d-1)}\left\{y^{d-2}\partial_x(f)[\psi_2,-] + x^{d-2}\partial_y(f)[\psi_1,-]\right\}$$
$$\cdot\ \theta_1\theta_2(\omega\otimes\Psi)$$

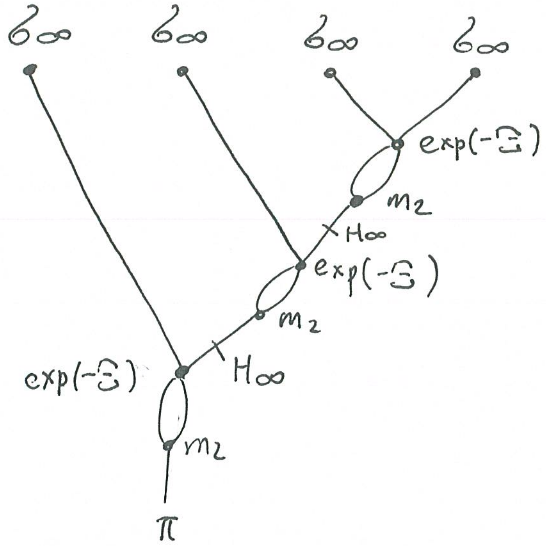
and (4.2) there

$$\exp(-\Xi) = 1 - [\psi_1,-]\otimes\theta_1^* - [\psi_2,-]\otimes\theta_2^* \qquad (2.3)$$
$$- [\psi_1,-][\psi_2,-]\otimes\theta_1^*\theta_2^*$$

For the usual reason, a diagram in (1.1) with left leg of final m_2 an internal edge has a zero contribution, so ⓑ, ⓓ, ⓒ do not contribute, leaving ⓐ, ⓔ.

ainfmf6

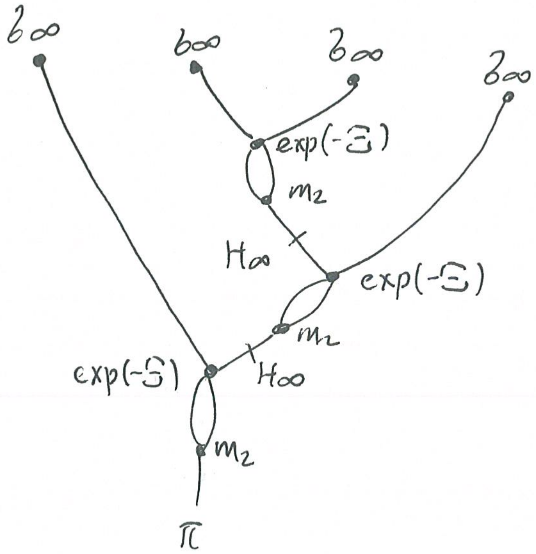
$b_4 =$



(a) (call this b_4^a)

(3.1)

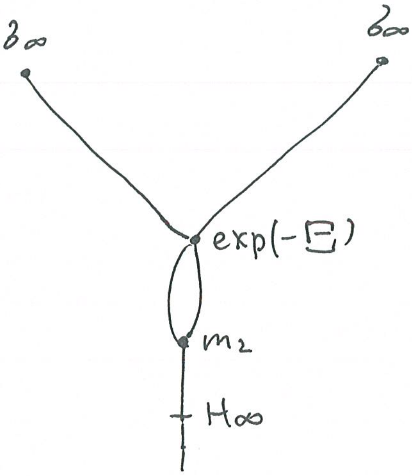
$+$



(e) (call this b_4^e)

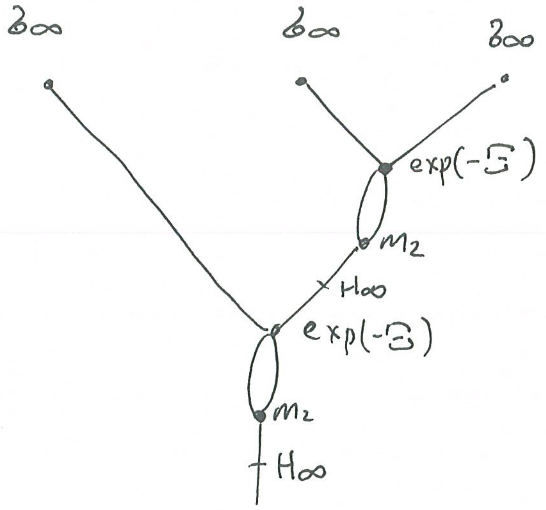
The computation of both diagrams begins with p. 16 of ainfmf5 where we computed

$T :=$



(3.2)

$S_1 :=$



(4.1)

Then

$$S_1(\Lambda_0 \otimes \Lambda_1 \otimes \Lambda_2) = H_\infty m_2 \exp(-\Xi)\Big(\Lambda_0 \otimes T(\Lambda_1 \otimes \Lambda_2)\Big)$$

$$= H_\infty m_2 \exp(-\Xi)\Big(-(-1)^{|\Lambda_1|}\Lambda_0 \otimes x^{d-3}\theta_1[\psi_1,\Lambda_1]\cdot[\psi_1,\Lambda_2]$$

$$+ (-1)^{|\Lambda_1|}\Lambda_0 \otimes y^{d-3}\theta_2[\psi_2,\Lambda_1]\cdot[\psi_2,\Lambda_2]$$

$$-(-1)^{|\Lambda_1|}\Lambda_0 \otimes x^{d-3}y^{d-2}\theta_1\theta_2[\psi_1,[\psi_2,\Lambda_1]]\cdot[\psi_1,\Lambda_2]$$

$$+\Lambda_0 \otimes x^{d-3}y^{d-2}\theta_1\theta_2[\psi_1,\Lambda_1]\cdot[\psi_1,[\psi_2,\Lambda_2]]$$

$$+\Lambda_0 \otimes x^{d-2}y^{d-3}\theta_1\theta_2[\psi_2,\Lambda_1]\cdot[\psi_1,[\psi_2,\Lambda_2]]$$

$$-(-1)^{|\Lambda_1|}\Lambda_0 \otimes x^{d-2}y^{d-3}\theta_1\theta_2[\psi_1,[\psi_2,\Lambda_1]]\cdot[\psi_2,\Lambda_2]\Big)$$

$$= H_8 m_2 \Big(1(\cdots)$$
$$- [\psi_1, -] \otimes \theta_1^* (\cdots) \quad (i)$$
$$- [\psi_2, -] \otimes \theta_2^* (\cdots) \quad (ii)$$
$$- [\psi_1, -][\psi_2, -] \otimes \theta_1^* \theta_2^* (\cdots) \quad (iii) \Big)$$

$$= H_\infty m_2 \Big(1(\cdots)$$

(5.1)

$$(i) \begin{cases} + (-1)^{|\Lambda_1|+|\Lambda_0|} [\psi_1, \Lambda_0] \otimes x^{d-3} [\psi_1, \Lambda_1] \cdot [\psi_1, \Lambda_2] \cdot \\ + (-1)^{|\Lambda_1|+|\Lambda_0|} [\psi_1, \Lambda_0] \otimes x^{d-3} y^{d-2} \theta_2 [\psi_1, [\psi_2, \Lambda_1]] \cdot [\psi_1, \Lambda_2] \cdot \\ - (-1)^{|\Lambda_0|} [\psi_1, \Lambda_0] \otimes x^{d-3} y^{d-2} \theta_2 [\psi_1, \Lambda_1] \cdot [\psi_1, [\psi_2, \Lambda_2]] \cdot \\ - (-1)^{|\Lambda_0|} [\psi_1, \Lambda_0] \otimes x^{d-2} y^{d-3} \theta_2 [\psi_2, \Lambda_1] \cdot [\psi_1, [\psi_2, \Lambda_2]] \cdot \\ + (-1)^{|\Lambda_0|+|\Lambda_1|} [\psi_1, \Lambda_0] \otimes x^{d-2} y^{d-3} \theta_2 [\psi_1, [\psi_2, \Lambda_1]] \cdot [\psi_2, \Lambda_2] \cdot \end{cases}$$

$$(ii) \begin{cases} - (-1)^{|\Lambda_1|+|\Lambda_0|} [\psi_2, \Lambda_0] \otimes y^{d-3} [\psi_2, \Lambda_1] \cdot [\psi_2, \Lambda_2] \cdot \\ - (-1)^{|\Lambda_1|+|\Lambda_0|} [\psi_2, \Lambda_0] \otimes x^{d-3} y^{d-2} \theta_1 [\psi_1, [\psi_2, \Lambda_1]] \cdot [\psi_1, \Lambda_2] \cdot \\ + (-1)^{|\Lambda_0|} [\psi_2, \Lambda_0] \otimes x^{d-3} y^{d-2} \theta_1 [\psi_1, \Lambda_1] \cdot [\psi_1, [\psi_2, \Lambda_2]] \cdot \\ + (-1)^{|\Lambda_0|} [\psi_2, \Lambda_0] \otimes x^{d-2} y^{d-3} \theta_1 [\psi_2, \Lambda_1] \cdot [\psi_1, [\psi_2, \Lambda_2]] \cdot \\ - (-1)^{|\Lambda_0|+|\Lambda_1|} [\psi_2, \Lambda_0] \otimes x^{d-2} y^{d-3} \theta_1 [\psi_1, [\psi_2, \Lambda_1]] \cdot [\psi_2, \Lambda_2] \end{cases}$$

$$\text{(iii)}\left\{\begin{aligned} &-(-1)^{|\Lambda_1|}[\psi_1,[\psi_2,\Lambda_0]]\otimes x^{d-3}y^{d-2}[\psi_1,[\psi_2,\Lambda_1]]\circ[\psi_1,\Lambda_2] \\ &+[\psi_1,[\psi_2,\Lambda_0]]\otimes x^{d-3}y^{d-2}[\psi_1,\Lambda_1]\circ[\psi_1,[\psi_2,\Lambda_2]] \\ &+[\psi_1,[\psi_2,\Lambda_0]]\otimes x^{d-2}y^{d-3}[\psi_2,\Lambda_1]\circ[\psi_1,[\psi_2,\Lambda_2]] \\ &-(-1)^{|\Lambda_1|}[\psi_1,[\psi_2,\Lambda_0]]\otimes x^{d-2}y^{d-3}[\psi_1,[\psi_2,\Lambda_1]]\circ[\psi_2,\Lambda_2]\Big) \end{aligned}\right. \qquad (6.1)$$

killed by H_∞

$$\begin{aligned} = H_\infty\Big(&\left.\begin{aligned} &-(-1)^{|\Lambda_0|+|\Lambda_1|}\theta_1 x^{d-3}\Lambda_0\circ[\psi_1,\Lambda_1]\circ[\psi_1,\Lambda_2] \\ &+(-1)^{|\Lambda_0|+|\Lambda_1|}\theta_2 y^{d-3}\Lambda_0\circ[\psi_2,\Lambda_1]\circ[\psi_2,\Lambda_2] \\ &-(-1)^{|\Lambda_1|}\theta_1\theta_2 x^{d-3}y^{d-2}\cancel{\Lambda_0}\circ[\psi_1,[\psi_2,\Lambda_1]]\circ[\psi_1,\Lambda_2] \\ &+\theta_1\theta_2 x^{d-3}y^{d-2}\Lambda_0\circ[\cancel{\psi_1},\Lambda_1]\circ[\psi_1,[\psi_2,\Lambda_2]] \\ &+\theta_1\theta_2 x^{d-2}y^{d-3}\Lambda_0\circ[\cancel{\psi_2},\Lambda_1]\circ[\psi_1,[\psi_2,\Lambda_2]] \\ &-(-1)^{|\Lambda_1|}\theta_1\theta_2 x^{d-2}y^{d-3}\Lambda_0\circ[\cancel{\psi_1},[\psi_2,\Lambda_1]]\circ[\psi_2,\Lambda_2] \end{aligned}\right\} \text{(0)} \\ &\left.\begin{aligned} &+(-1)^{|\Lambda_0|+|\Lambda_1|}x^{d-3}[\psi_1,\Lambda_0]\circ[\psi_1,\Lambda_1]\circ[\psi_1,\Lambda_2] \\ &-(-1)^{|\Lambda_1|}\theta_2 x^{d-3}y^{d-2}[\psi_1,\Lambda_0]\circ[\psi_1,[\psi_2,\Lambda_1]]\circ[\psi_1,\Lambda_2] \\ &+\theta_2 x^{d-3}y^{d-2}[\psi_1,\Lambda_0]\circ[\psi_1,\Lambda_1]\circ[\psi_1,[\psi_2,\Lambda_2]] \\ &+\theta_2 x^{d-2}y^{d-3}[\psi_1,\Lambda_0]\circ[\psi_2,\Lambda_1]\circ[\psi_1,[\psi_2,\Lambda_2]] \\ &+(-1)^{|\Lambda_1|}\theta_2 x^{d-2}y^{d-3}[\psi_1,\Lambda_0]\circ[\psi_1,[\psi_2,\Lambda_1]]\circ[\psi_2,\Lambda_2] \end{aligned}\right\} \text{(i)} \end{aligned}$$

ainfmf6

$$
\left.\begin{aligned}
&-(-1)^{|\Lambda_0|+|\Lambda_1|}\, y^{d-3}\,[\psi_2,\Lambda_0]\cdot[\psi_2,\Lambda_1]\cdot[\psi_2,\Lambda_2]\\
&+(-1)^{|\Lambda_1|}\,\theta_1 x^{d-3}y^{d-2}\,[\psi_2,\Lambda_0]\cdot[\psi_1,[\psi_2,\Lambda_1]]\cdot[\psi_1,\Lambda_2]\\
&-\theta_1 x^{d-3}y^{d-2}\,[\psi_2,\Lambda_0]\cdot[\psi_1,\Lambda_1]\cdot[\psi_1,[\psi_2,\Lambda_2]]\\
&-\theta_1 x^{d-2}y^{d-3}\,[\psi_2,\Lambda_0]\cdot[\psi_2,\Lambda_1]\cdot[\psi_1,[\psi_2,\Lambda_2]]\\
&+(-1)^{|\Lambda_1|}\,\theta_1 x^{d-2}y^{d-3}\,[\psi_2,\Lambda_0]\cdot[\psi_1,[\psi_2,\Lambda_1]]\cdot[\psi_2,\Lambda_2]
\end{aligned}\right\}\ \text{ii} \qquad (7.1)
$$

$$
\left.\begin{aligned}
&-(-1)^{|\Lambda_1|}\, x^{d-3}y^{d-2}\,[\psi_1,[\psi_2,\Lambda_0]]\cdot[\psi_1,[\psi_2,\Lambda_1]]\cdot[\psi_1,\Lambda_2]\\
&+x^{d-3}y^{d-2}\,[\psi_1,[\psi_2,\Lambda_0]]\cdot[\psi_1,\Lambda_1]\cdot[\psi_1,[\psi_2,\Lambda_2]]\\
&+x^{d-2}y^{d-3}\,[\psi_1,[\psi_2,\Lambda_0]]\cdot[\psi_2,\Lambda_1]\cdot[\psi_1,[\psi_2,\Lambda_2]]\\
&-(-1)^{|\Lambda_1|}\, x^{d-2}y^{d-3}\,[\psi_1,[\psi_2,\Lambda_0]]\cdot[\psi_1,[\psi_2,\Lambda_1]]\cdot[\psi_2,\Lambda_2]
\end{aligned}\right\}\ \text{iii}
$$

Let us denote this by $H_\infty(\cdots)$. Then b_4^a from (3.1) is

$$b_4^a(\Lambda\otimes\Lambda_0\otimes\Lambda_1\otimes\Lambda_2) = \pi m_2 \exp(-\Xi)\big(\delta_\infty(\Lambda)\otimes H_\infty(\cdots)\big)$$

$$= \pi m_2 \exp(-\Xi)\big(\Lambda\otimes H_\infty(\cdots)\big)$$

1 here pair with <u>nothing</u> (H_∞ has θ-degree >0 so all gets killed by π)

θ_j^* pair with θ_j terms in $\Lambda\otimes H_\infty(\cdots)$ which come from constant terms in $\cdots$

$\theta_1^*\theta_2^*$ pair with θ_1 and θ_2 terms in $\cdots$

$$= \pi m_2 \exp(-\Xi)\big(\Lambda \otimes H_\infty(⓪ + ① + ⓘⓘ + ⓘⓘⓘ)\big)$$

Now $H_\infty \sim \partial_x(-)\mathcal{O}_1 + \partial_y(-)\mathcal{O}_2$ so vanishes on terms like $\mathcal{O}_1 x^{d-3}$. So $H_\infty(⓪) = 0$

Note At this point we decided to go attack the general case!

Note There are sign errors here because of the issues discussed in ainfmf14, but we will not correct these as the approach in this note is a dead end.