Note on morphisms of theories

To expand on my answer to Sam's question at the end of lecture #2, here is an example. Recall for first-order geometric theories T, T' a candidate def^N for $\mathrm{Hom}(T, T')$ is given by topos theory as follows

$$\mathrm{Hom}(T, T') := \underline{\mathrm{Hom}}(\mathcal{B}(T'), \mathcal{B}(T))$$
$$\cong \{\text{models of } T \text{ in } \mathcal{B}(T')\}.$$

I argued that the topos theoretic approach may provide surprising examples of morphisms $T \longrightarrow T'$ that might not appear natural from a purely logical point of view. Here is one such: let $T = \mathcal{G}rp$, the theory of groups, and $T' = \mathcal{L}oc\mathcal{R}ng$, the theory of local rings. Then since $\mathcal{B}(T')$ is the Zariski topos $\mathcal{Z}ar$, the above calculation gives

$$\mathrm{Hom}(\mathcal{G}rp, \mathcal{L}oc\mathcal{R}ng) \cong \{\text{group objects in } \mathcal{Z}ar\}$$

But group objects in $\mathcal{Z}ar$ arise in algebraic geometry: an <u>algebraic group</u> (e.g. $GL(n, \mathbb{C})$) is a group object in Schemes over $\mathbb{C}$, and in particular in $\mathcal{Z}ar$, so every algebraic group gives rise to a morphism $\mathcal{G}rp \longrightarrow \mathcal{L}oc\mathcal{R}ng$. This doesn't seem obvious without knowing $\mathcal{B}(\mathcal{L}oc\mathcal{R}ng) \cong \mathcal{Z}ar$, but maybe there is a purely logical way to see it?