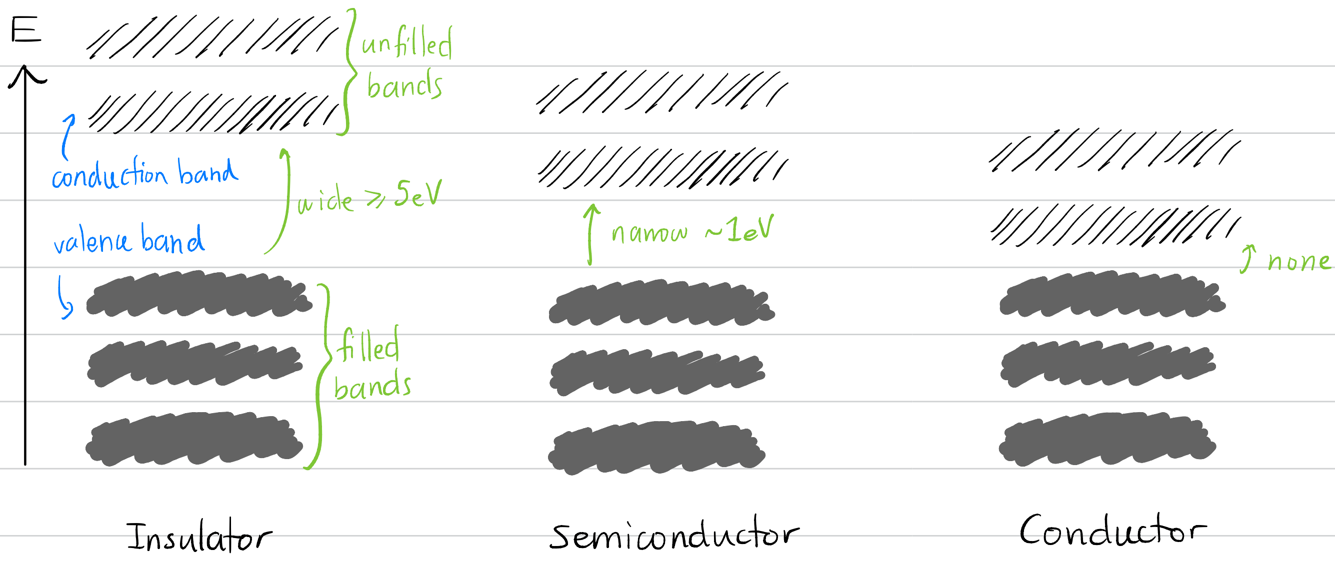
Singular Learning Theory 21 - Solid state physics

Solid state physics is a branch of condensed matter physics, which studies phases of matter with many strongly interacting components: for example crystals (or crystalline solids). The precise nature of the periodicity in a crystal affects its electrical, magnetic, optical and mechanical properties, and an understanding of this relationship (between microscopic symmetry and macroscopic bulk properties of a material) is often used for engineering purposes.

The applications to engineering sometimes involve exploiting an existing material found in Nature, but increasingly also designing materials with desirable properties by understanding the link between the microscopic symmetry and macroscopic behaviour and engineering the former [N].

Example 1 The standard classification of materials into conductors (low resistance) insulators (high resistance) and semi-conductors (intermediate resistance). The resistance of a semi-conductor can be designed by introducing impurities in a process called doping; even a few impurities (on the order of one in a million) can significantly change the electrical properties.



Here we see how

Crystal structure → structure of energy bands → bulk electrical properties

(to be clear, not about point symmetry)

Example 2 (Twisted bilayer graphene) Graphene is a hexagonal lattice made of Carbon atoms. In 2018 it was discovered that two (one atom) layers of graphene stacked with their lattices offset by 1.1° is a superconductor at up to 1.7K, the so called "magic angle". Normally graphene is an insulator. This was a big deal.

zero resistance

The abstracts of [C], [Y] contain terms like

- At 1.1° the electronic band structure of TBG exhibits flat bands near zero Fermi energy... quantum oscillations in the longitudinal resistance indicate the presence of small Fermi surfaces near the correlated insulating state

- One such feature is the van Hove Singularity (VHS) in Moiré bands... Generally speaking VHS with divergent density of states (DOS) in 2D systems are associated with saddle points of energy dispersion in $\underline{k}$ space. When a VHS is close to Fermi energy, the increased DOS amplifies electron correlation, resulting in ... superconductivity at low T.

The aim of this talk and the sequel are to explain these terms, and the analogy between SLT and solid state physics.

[Insert here §1 of SLT12]

The dispersion relation is the function $E = E(\underline{k})$ giving the energy of a particle (here an electron) in terms of its momentum. The density of states we have explained. The Fermi energy is the energy of the topmost filled level in the ground state. The Fermi-Dirac distribution [K, p.137] gives the probability that an orbital at energy ϵ will be occupied in an ideal electron gas in thermal equilibrium

$$f(\epsilon) = \frac{1}{\exp((\epsilon-\mu)/k_B T)+1}$$

In the limit $T \longrightarrow 0$, we have

$$f(\epsilon) \longrightarrow \begin{cases} 1 & \epsilon < \mu \\ 0 & \epsilon > \mu \end{cases}$$

That is, at zero temperature the states below μ (the Fermi energy) are filled. In momentum space these states $\{ \underline{k} \mid E(\underline{k}) \leq \mu \}$ fill up a ball, the surface of which is called the Fermi surface. The linear response of a crystal to an electric, magnetic or thermal gradient is determined by the "shape" of the Fermi surface (topology and geometry) because currents are due to changes in the occupancy of states near the Fermi energy.

That is, the Fermi surface is a level set of the energy function E.

⌈ Recall in SLT

$$\lambda = \lim_{t \to 0} \frac{\log(V(at)/V(t))}{\log a} \qquad V(t) = \int_{K(w)<t} \varphi(w)\,dw$$

$a > 0,\ a \neq 1$ ⌋

scaling exp. of DOS

DOS.

5|7|21

Explaining semiconductors

References

[K] C. Kittel "Introduction to solid state physics" 8th edition

[N] M. Nielsen "Maps of matter" Blog post 2021.

[C] Y. Cao et al "Unconventional superconductivity in magic-angle graphene superlattices" Nature 2018.

[Y] N. Yuan et al "Magic of high-order van Hove singularity" 2019